

ENERGY, PARTICLE ACCELERATORS AND SYNCHROTRON RADIATION

$E=mc^2$

The simple relation $E=mc^2$ is clearly a valid one, demonstrated most notably over Hiroshima in 1945 while bringing about the end to the second world war, and more commonly in the nuclear power plants supplying major cities. Though Einstein was not awarded the Nobel prize for this discovery, it is the one for which the public remembers him most. But isn't the relation $E=mc^2$ a consequence of the special theory of relativity? We have disposed of length contraction and time dilation due to velocity, yet it appears the radiation continuum model must cling to this king of all relativity results. Clearly the RCM must support this facet of relativity theory to be of any validity. We are going to show in actuality that, as with the gravitational red-shift and the slowing of clocks, $E=mc^2$ is not actually a consequence of relativity, but may be derived directly by studying the absorption and emission of photons.

In any inertial frame of reference, a photon has a constant detectable velocity of c . Energy can be expressed as a change in momentum times velocity. In the case of a photon in a given inertial frame then, we can denote the energy of a photon as its momentum times c , since c is the observed speed of light in all inertial frames of reference. Now, the momentum of an object is simply its mass times its velocity. In the case of a photon in a given reference frame, its momentum is then its mass times c . Combining these two relations then yields the result that a photon's energy is given simply by its mass times c^2 .

Thus we have shown that a photon, a massless entity, has an "effective mass" equal to its energy divided by c^2 . But this is still a far cry from stating that all ponderable mass has an energy associated with it of mc^2 . Consider the case of photons bombarding a material. Each photon that is absorbed transfers its energy to the material, by raising an electron to a higher energy level. The material subsequently releases an equivalent amount of energy when that electron returns to a lower state. If we could convert some or all of the mass in a material into radiant energy, we would expect to see much more energy released than would be obtained by, say, burning the material. The energy release due to burning is simply associated with converting the form of the material, by breaking the covalent or ionic bonds holding the constituent molecules together. This process does not actually annihilate the material, as would a change from matter to radiant energy. Let us return to the case of photons bombarding a material.

Imagine an object with a specific mass and energy stationary in our reference frame, as depicted in figure 7-1. Let this object be struck simultaneously by two photons, striking it from above and below as in the figure. Each of these photons has a specific energy, which can be expressed as an equivalent mass as explained above. Since the velocity of the object is zero before the photon collisions, its kinetic energy is also zero. After the symmetric, simultaneous collisions of the photons, the kinetic energy of this mass is still zero, since it was struck with equal force from each side. Now, let us assume that each of the photons was absorbed by the object, each by raising an electron in one of the atoms of the mass to a higher energy state. In this case, we have an example of a perfectly inelastic collision. This means that the photons do not "bounce off" the mass and carry their kinetic energy away with them, as in the case of two pool balls striking each other. The total energy of the object is now equal to its original energy plus the energy of the two photons. However, since the photons have been completely absorbed by the electrons, we can alternatively state that the mass of the object has increased by the "equivalent mass" of the two photons.



Figure 7-1 The mass of a block absorbing two photons will increase by the mass equivalent of the energy of the photons.

If we let the energy of each photon be denoted by e , then the *change* in energy of the object is equal to $2e$, and the *change* in mass is given by $2e$ divided by c^2 . Taking the ratio of these two quantities yields the resultant change in mass of a material as a result of absorbing radiant energy--the change in mass is equal to the change in energy divide by c^2 . Thus absorbing a unit of radiant energy results in an increase in mass equal to the energy absorbed divided by c^2 . Likewise, the conversion of mass *into* radiant energy would result in the energy produced being equal to the mass converted times c^2 . If we converted the entire mass of a material into energy, then the change in mass would equal the total mass of the object, and the energy released would equal the total energy of the object. If we represent the total energy of the object by E , and the total mass of the object by m , we are left with the relation $E = mc^2$.

This is, of course, the result we are looking for. Note that, as in the case of the gravitational potential, the energy released does not necessarily need to be in the form of photons. The universe tends to treat all forms of energy as equivalent. What is important in this relation is that we are converting mass to energy, not simply accelerating a mass to a particular velocity to produce kinetic energy. As a body is accelerated, it retains its mass energy, and acquires the kinetic energy.

The kinetic energy of a moving body, as derived in standard Newtonian mechanics, is equal to one-half the mass of the object times the square of its velocity. Thus the total energy of a moving body is given by its mass times c^2 plus one-half its mass times its velocity squared. If we express this energy as an effective equivalent mass, then, as long as the velocities are kept well below the speed of light, it can be shown that the effective mass of a moving body is equal to the gamma factor times the actual mass of the object. This is the familiar term for relativistic mass increase due to velocity, applicable for velocities much less than c , which accounts for all interactions of macroscopic items for which the rest mass and velocity are accurately known. Bear in mind again that this effective mass increase is due to the kinetic energy of a ponderable mass, and is not actually conversion of the energy into ponderable mass, which is impossible except for processes such as nuclear fusion or fission. This is not a subtle distinction. In relativity, the actual mass of the object is presumed to increase by the factor γ , such that, at the speed of light, the mass of the object would be infinite. Additionally, conversion of this light speed mass to energy would release infinite energy. If this were the case, then, if we could somehow accelerate an electron and a positron to light speed, allowing them to annihilate each other, that one reaction would provide us with an infinite source of energy. Unfortunately, in the relativistic model, it would also take infinite energy to accelerate anything even so small as an electron to light speed, so this cannot happen.

In the RCM theory, this problem does not occur. The effective mass equivalent expressed for the moving object is simply that, not an actual change in mass. Furthermore, the approximation to γ times the mass applies only at speeds much less than c . Once the velocity approaches c , that approximation is no longer valid, and the effective mass equivalent is much less than that predicted by relativity theory. Thus, even at the speed of c , where relativity predicts an infinite mass, the effective mass of the object in RCM theory is only one and one-half times its rest mass (as we will see later, if we include the additional kinetic energy of the mass equivalent of the kinetic energy itself, we wind up with an effective mass of two times the object's rest mass). Furthermore, to accelerate such an object to light speed, one must accelerate only the object's rest mass. The effective mass equivalent is an effect of the velocity, not an addition to the mass which must also be accelerated. Actually, we might have to provide additional energy to accelerate the effective mass of the object's kinetic energy as well, in the same manner as gravity has to act on Mercury's kinetic energy. But, even at the speed of c , this would be equivalent to accelerating a mass of at most two times the initial mass, not an infinite mass as predicted by Einstein. Thus to accelerate an object to higher and higher speeds, even those approaching c , requires only a continuous application of a finite amount of force. If enough fuel can be provided to apply the force for a long enough time, an object will eventually reach, and then exceed, the speed of c .

So what happens in particle accelerators? In these devices, elementary particles are routinely accelerated to velocities approaching c , and their mass apparently increases by the factor γ , not by the smaller effective mass predicted by RCM theory. The answer to this question lies not in an actual change in mass with velocity, but rather in the way in which mass is determined in a particle accelerator.

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MASS AND THE GAMMA FACTOR

The relativistic theory of space-time suggests that, as an object's velocity increases, so does its mass. As you might expect, this mass increase is proportional to the factor gamma, γ , which as we have seen is equal to one over the square root of one minus the square of the velocity of the object, with this velocity expressed as a fraction of c . At the speeds with which we and our man-made machines are accustomed to traveling, this factor is imperceptible. However, at speeds approaching c , the effect of multiplying the mass by this number becomes great, and the mass of an object in motion becomes many times greater than its "rest" mass. This apparent effect has been observed experimentally in the case of elementary particles, including electrons, which we will consider for this discussion.

Suppose we wish to force an electron of a given mass to move in a circle by applying a constant radially inward force. In order to cause an object to deviate from a straight path, a force must be applied which accelerates that object in the required direction. Thus, moving an object in a circle of a given radius requires a constant acceleration of a fixed value. The force required to produce a given acceleration is equal to the mass of the object times the acceleration desired. It so happens with electrons and other elementary particles in motion that the only appropriate way to determine their mass is to place them in the presence of an external force and see what acceleration is caused by the application of that force. Thus, if, as an electron goes faster and faster, a greater and greater force is required to produce a given acceleration, it can be argued that its mass has increased. Frame A of figure 7-2 depicts a stationary electron in the presence of a magnetic field between two parallel plates. An electron in the presence of a magnetic field experiences a force in a direction perpendicular to the direction of the field. The direction of the magnetic field in frame A is through the plane of the page, resulting in a force on the electron from the lower plate to the top plate as pictured. The magnetic field is not shown, but just the resultant force, for simplicity of visualization. The magnetic field produces a defined force on the electron, due to its negative electric charge and its motion, in a direction perpendicular to both the motion of the charge and the direction of the magnetic field.

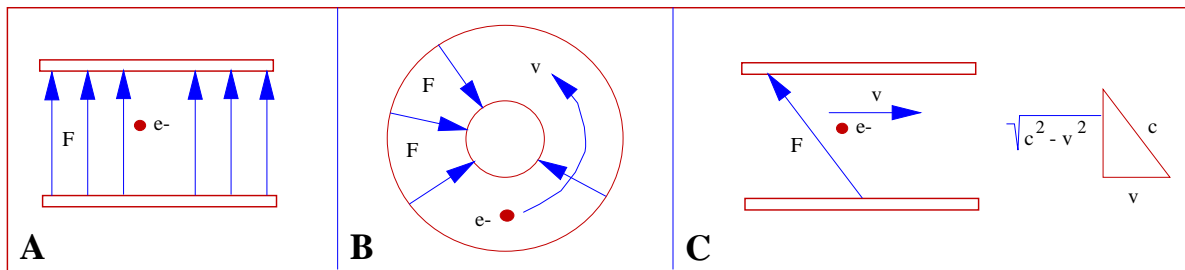


Figure 7-2 A moving electron experiences less force due to a magnetic field than does a stationary electron. The reduced force appears as an increase in the mass of the electron.

If the force were to act on the electron for a short time, the electron would be accelerated toward the top plate very quickly, and the electron would strike that plate. In frame B, the plates have been curved into a circle, such that at all points the force on the electron is radially inward. If the electron is placed in motion, it will have a tendency to continue in a straight line, however, if the magnetic field (still through the plane of the page at all points) is set to the correct value, the force of the electron trying to go straight will be exactly balanced by the force pushing the electron toward the inside plate, and the electron will travel in a circle. For a specific radius and electron velocity, the acceleration required is clearly defined. As an example, the acceleration due to gravity is well defined, thus it can be calculated that the radius of orbit of a satellite which is to have a velocity which keeps it continuously over the same point on Earth is about 22,500 miles. The force on the satellite is equal to its mass times the

acceleration due to gravity. If there were no gravity, we would have to create a force which would draw the satellite toward the Earth with the same acceleration which gravity produces.

In frame B, if the electron is moving with a fixed velocity, the force (magnetic field) required to keep it moving in a circle increases as the velocity increases by the factor γ over what it should be to produce the required acceleration according to Newtonian mechanics. Now, since the acceleration can be expressed as the force divided by the mass of the object, and the required force has increased over what one would expect to use, it would appear that the mass of the electron has also increased by the same factor of γ . At varying velocities for the electron, the results are always the same, the force required is increased by γ for that velocity, and therefore the mass of the electron appears to have increased by γ as well. This is certainly the argument used in relativity theory, and the "observed" increase in mass in particle accelerators is considered as one of the major factors in support of the theory.

Now let's look at things from the electron's perspective. Recalling that electromagnetic radiation travels at a velocity of c in the detector's frame of reference, then the lines of force between the two plates can be thought of as "moving" at a velocity of c from the bottom to the top plate (or outside to inside plate). Note that the force does not actually move, but it is helpful to provide a graphic explanation of the ratio of magnitude and direction of forces, so that we can see where the gamma factor arises. In frame C we see a small section of the circular plates from the point of view of the electron. In its frame of reference, it is standing still, and the lines of force produced by the magnetic field are traveling, not only directly across its path, but also backwards at a velocity (there's that unfortunate word again) equal to what we in the reference frame of the lab call the velocity of the electron. This results in a triangle with a hypotenuse of c , and a leg across the path of the electron given by the Pythagorean theorem as the square root of c^2 minus the square of the electron's velocity. Now, in the same way that velocities were described earlier as being made up of components at right angles to each other, so it is with forces. Thus a force along the line represented by c can be considered as being made up of two components, one along the line of motion of the electron, and the other along the line perpendicular to the two plates. It is the value of this component of force which the electron feels accelerating it toward the top plate. In the case of this force acting across the path of the electron, the resultant force is less than the initial force by one over gamma. If we wish for this resultant force to have the value required to keep the electron moving in the desired circular path, we would have to increase the applied force by the factor gamma. This required increase in the initial applied force comes about without an increase in the mass of the electron. In fact, if there *were* an increase in the mass of the electron, the initial force would have to be increased even further, by a total factor of γ^2 to account for this.

Note that it is the nature of this force--an electromagnetic one--which requires that the lines of force be "moving" past the electron at the speed of c . If the force were from a piston, or a blast of air, then its effect would not depend on the forward velocity of the object being forced. But, as we have seen earlier, Maxwell's equations require that electromagnetic interactions occur at the speed of c with respect to the observer (in this case the electron). While the above discussion indicated that it was the resultant force which was scaled by the gamma factor, it can be alternately viewed that the magnetic field is skewed instead, and the resultant cross-product of the electron's charge and velocity properties with the gamma scaled magnetic field produces a smaller force on the electron--smaller by the same factor gamma. The most important part of this discussion is the realization that electromagnetic interactions do not behave in the same manner as purely physical forces. A force applied across the path of a moving softball and a stationary softball will produce the same acceleration in each along the line of that force. In the case of a moving electron in a magnetic field, the initial velocity of the electron across the path of the magnetic field will cause the value of that magnetic field's effect to be skewed by the factor γ . This happens because the requirement that electromagnetic effects operate at the speed of c in the observer's frame of reference must be satisfied. In actuality, of course, all physical effects are attributable to electromagnetic interactions. Thus if we tried to hit a baseball traveling across our path at a velocity approaching c , in addition to needing a very good eye, we would expect that we should require a much larger force to change its direction.

Thus, in our experience, no large physical objects have been moved and measured at a great enough velocity to determine the relativistic effects on mass. The elementary particles which do move fast enough can have their mass determined only by means of electromagnetic interactions. According to the radiation continuum model, these particles do not suffer an increase in mass proportional to their velocity, but only a decrease in the electromagnetic force which is proportional to γ . Since electromagnetic means are required to make such determinations as the mass of moving particles, these "measurements" are necessarily scaled by the same factor as the electromagnetic forces

themselves, resulting in the apparent increase in mass of particles in motion. We can continue the analysis of an electron forced to move in a path defined by a series of magnets by considering devices known as synchrotrons, which are the subject of the next section.

SYNCHROTRON RADIATION

If a charged particle, say, an electron, is confined to move in a circular path by a magnetic field, the continual change in direction experienced by the electron will produce an electromagnetic wave. This is because a moving charge produces an electric current, and a changing electric current produces electromagnetic waves--light. A simplified schematic of this type of device is depicted in figure 7-3. In this case, it is the changing direction of the electron's motion which results in a changing current. The frequency of the light produced would be, it seems, equal to the rate of revolution of the electron about its circular path, or, for an electron traveling near "light speed", approximately c divided by the circumference of the path. In reality, the observed frequency of the light is much greater, and increases with the velocity of the electron by about the cube of the gamma factor, or γ^3 . While the relativistic viewpoint explains this phenomenon with relativistic aberration and time dilation, the radiation continuum model provides a much simpler approach. We will first consider the relativistic explanation.

We begin by looking at two sources of synchrotron radiation. The case described above is an oversimplified picture of an actual synchrotron. In such a device, the electrons are confined to a circular orbit, but the radius of the actual device does not equal the radius of curvature caused by the bending magnets. There are several straight sections between the magnets, so that the actual device is much larger than the radius of each magnet. This is analogous to taking twelve curved sections of model railroad track, which can be joined together to make a circle of a given radius. If you now place one piece of straight track between each two curved pieces, the resulting circumference will be roughly twice the original layout. In a synchrotron, the radius value of interest is the radius of the bending magnet, which would equate to the radius of just the curved track sections. For this reason, we will diagram the synchrotron as a perfectly circular device, absent of any straight sections, so that the radius of the synchrotron equals the radius of the bending magnets, as in figure 7-3.

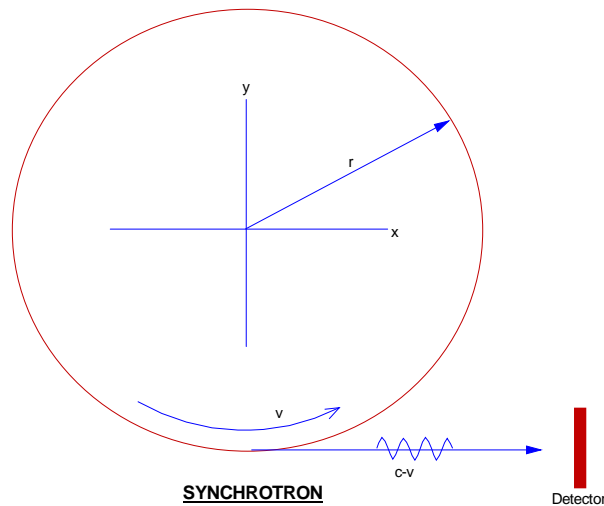


Figure 7-3 An electron confined to a circular orbit will emit light in a direction tangential to that orbit at a frequency proportional to γ^3 for the electron's velocity.

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The radiation emitted from a synchrotron is a very narrow beam, given off in a direction tangent to the orbit and in the direction of motion of the electron. Relativistic theory states that relativistic aberration causes an increase in the frequency by about γ . In addition to this, since the emitting electron is moving in the direction of the emission, the radiation reaches the observer in a much shorter time period than was required to emit the radiation. This Doppler type effect increases the frequency by about a factor of γ^2 , so that altogether the observed radiation is increased from the frequency of the electron's orbit by a factor of roughly γ^3 .

In particular, the radiation from a synchrotron is defined by its critical wavelength. The critical wavelength defines that frequency which divides the total radiated spectrum in half. One half of the radiated power is below that frequency, and one half the power is above that frequency. The peak power of a synchrotron is radiated at a wavelength equal to seven-tenths of the critical wavelength, and almost all the power is radiated between two-tenths and ten times this wavelength. The radiated spectrum is very broadband, as it contains the transformed frequency and multiple harmonics. The harmonics are separated on the order of one megahertz (one million cycles per second), which, given the high radiated frequency, are difficult to individually discern. Thus the total radiation appears as a continuous band of radiation, spread evenly in power around the critical wavelength. For this discussion, we are interested in the peak power of the radiation, which occurs at seventy percent of the critical wavelength. We will define the initial frequency as being equal to the rate of revolution of the electron's orbit around its circular path, given approximately by c divided by the circumference of the circle. From the geometry of the figure and the gamma cubed relation, the critical frequency may be calculated for any synchrotron. In terms of the initial frequency, the critical frequency is equal to two times γ^3 times the initial frequency.

A second type of synchrotron radiation source is known as either a wiggler or an undulator. These devices are similar except that a wiggler perturbs the electron's path to a greater extent than an undulator. We will consider the case of an undulator, as it is simpler to analyze in the general case. In an undulator, a fast moving electron in a straight section of trajectory is forced to move in a periodic fashion for a short distance before returning to its original course. In figure 7-4, a series of five magnets cause the electron to follow the periodic path shown, before returning to its original trajectory. The direction of the magnetic field is alternating into and out of the plane of the paper with each magnet. A series of three magnets produces one period of oscillation, and is called a single-period wiggler or a wavelength shifter. The distance between any three magnets is therefore also the periodic wavelength of the electron's path, and we could state that the initial frequency of the electron is roughly c divided by this wavelength. If the field of the magnets is small so as to only cause a minor deviation from the straight path, the device is called an undulator. In this case, the forward velocity of the electron does not deviate substantially from its initial velocity. The radiation here is caused by changing the current of the electron by making it follow the periodic path. In this type of radiation source, the radiation detected is proportional to the square of γ . The radiation from an undulator is not broadband as in the case of the synchrotron, but is centered very tightly around the fundamental frequency of the device. The relativistic argument for the shift in frequency is as follows. In the reference frame of the experimental setup, the electron, traveling at a velocity close to c , completes one period of the oscillation at a frequency of c divided by the wavelength of the electron's path. Now, in a frame moving along the apparatus at a velocity equal to that of the electron, the frequency is skewed by the factor of γ , due to Lorentz contraction. The moving electron thus emits radiation equal to its frequency of oscillation in its frame of reference. To determine the detected frequency in the reference frame of the laboratory, we must apply the relativistic Doppler shift which, to first order, results in a frequency shift of two times γ^2 from the initial frequency of the electron's oscillations in its path. This is the frequency of interest in an undulator, and is the one we will derive utilizing the radiation continuum model. We will begin with this derivation, and then consider the case of the synchrotron.

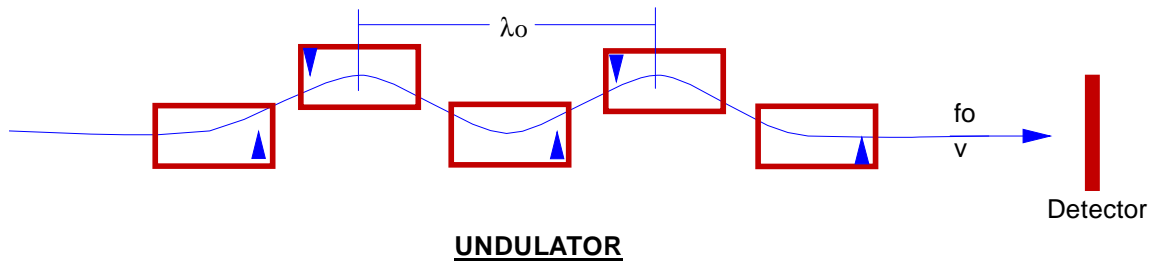


Figure 7-4 An electron forced to move in a periodic fashion along its otherwise straight path will emit a frequency proportional to γ^2 .

In the undulator, we have a simple case of a source of light in motion with respect to an observer. While in the past we have imagined an explosion, a headlight, a star or a radio emitter, the effect here is the same. If we were moving along the axis of the undulator at the same speed as the electron, we would see an electron moving up and down in a periodic fashion with a frequency of c divided by the periodic wavelength of the device. This motion would produce a light at this frequency, which we would see radiating ahead of us. Now, an observer stationary in the lab frame would appear to be moving toward us at a velocity equal to the speed of the electron through the undulator. This observer would require that component of the radiation leaving our source at a velocity of c minus the electron's velocity, and his detected frequency would thus be equal to the initial frequency times c divided by this component velocity. Using the knowledge that the velocity of the electron is close to c , it is simple to show that this relation for the shift in frequency observed in the laboratory frame is equal to two times γ^2 , as in the relativistic model.

In order to analyze the synchrotron, we must consider the frame of the electron a little more closely. In the undulator, the electron is moving at a certain fixed speed, and thus its clocks would slow such that internally it appears to be oscillating at a frequency shifted to the red by γ for that velocity. Thus the electron will emit radiation at that higher frequency as measured by its clocks. However, since it is moving, and we are aware that it is moving, the transformed frequency in our frame, as measured by our clocks, is still the initial frequency, the frequency at which we observe the electron tracing its path through the undulator.

In the synchrotron, the case is different. Here the oscillations are due to the electron moving in a circle (in the idealized case). Thus the electron is moving at a fixed velocity, but is also standing stationary in the laboratory frame. In figure 7-3, the oscillation can be pictured as due to the motions of the electron in the y or vertical axis only, since, over the course of one complete revolution, the electron would begin and end at the same x coordinate, with x being considered the axis along which the radiation will be viewed. In this system, the frequency which the electron is emitting must take into account the slowing of its clocks in its frame. Thus, the emitted frequency in the electron's reference frame is equal to γ times the initial frequency. This radiation can be thought of as that frequency measured with "local" clocks, which is of course higher than the frequency measured with "proper" lab-based clocks. Now, in the laboratory frame, we have an electron emitting radiation at a frequency of γ times the initial frequency, and moving along the x axis at a velocity close to c . From here out the analysis is the same as for that of an undulator, or any moving source, except that the initial frequency is already skewed one time by the gamma factor. Thus the observed peak power frequency of the synchrotron is roughly equal to two times γ^3 times the initial frequency. This is equivalent to the expression obtained for the frequency of peak power derived earlier using the relativistic model.

The table below lists several synchrotron sources, their energy, E , in trillions of electron-volts (GeV), and the critical wavelength, λ_c in angstroms (10^{-10} m). The velocity of the electron can be determined by knowing γ , which is equal to 1,957 times the energy. The bending magnet radius, r , in meters, is equal to the critical wavelength times E^3 divided by 5.6, and is included in the table. The relativistic peak power wavelength is given by .71 times λ_c . The radiation continuum value for the peak power wavelength is derived according to the above discussions.

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These two wavelengths, relativistic and RCM, are provided in the table, expressed in units of angstroms. The table shows the close correlation between the two values, even with the simplifying assumptions made in the above derivations. The source for the above table is HANDBOOK ON SYNCHROTRON RADIATION, Vol 2, Marr, North-Holland Physics Publishing, 1987.

Location	Energy (GeV)	$\lambda_c(\text{\AA})$	γ	r(m)	$.71\lambda_c(\text{\AA})$	$\lambda_{rc}(\text{\AA})$
Beijing - BEPC	2.80	2.60	5480	10.19	1.85	1.95
France - SUPERACO	0.80	18.50	1566	1.69	13.14	13.85
Italy - ADONE	1.50	8.30	2936	5.00	5.89	6.21
Japan - UVSOR	0.75	29.00	1468	2.18	20.59	21.71
Hamburg - HASYLAB	3.70	1.34	7241	12.12	0.95	1.00
Japan - TERAS	0.60	10.00	1174	0.39	7.10	7.48

Table 7-1 *A comparison of the critical wavelengths of various synchrotron sources as computed using standard relativistic procedures and RCM theory.*

PHOTON ENERGY AND THE NATURE OF PHOTONS

Max Planck determined that the energy of a photon must appear in packets which he called quanta. He made this discovery in trying to derive a model for the radiation emitted from a black-box at any given temperature. The classical explanation for such an effect would have indicated that there would be more energy produced at high frequencies, such as the x-ray band, and also that the box would effectively contain infinite energy. If the first conclusion were correct, we could all expect to be blinded from cooking with a cast iron skillet, while the second conclusion is clearly ridiculous. The second conclusion, in fact, resulted in the common name for this problem, the ultraviolet catastrophe.

Planck was able to derive an expression which matched exactly the actual distribution of energy emitted from a black-body. From this relation, without actually describing the nature of these quanta, he was able to show that each quanta has an energy equal to a small constant times the frequency. This small constant is now known as Planck's constant, and is usually denoted by the letter h. We generally refer to these quanta as photons, and say that a given photon carries energy equal to h times its frequency.

Einstein took Planck's quanta concept and applied it to another interesting problem of the times--a phenomenon known as the photo-electric effect. If we shine a light of a given frequency on a metal, some electrons will be emitted from the surface of the metal, and each of these electrons will have a specific energy. If the intensity of the light is increased, rather than getting higher energy electrons, we simply increase the number of electrons emitted from the metal. Only by increasing the frequency of the incident light do we increase the energy of the electrons emitted. Utilizing Planck's relation between frequency and energy, it is fairly easy to explain this effect. A certain amount of energy is required to knock an electron loose from the atom it is contained in. Light frequencies associated with energies below this amount will not knock out any electrons. Light above this frequency will cause the metal to emit an electron. The kinetic energy of the emitted electron is the directly proportional to how much higher the energy, or frequency, of the incident light is above this critical value. The higher the frequency, the more energetic the electron. Increasing the intensity of the light increases the number of quanta, or photons, striking the metal, and therefore increases the number, but not the energy, of the emitted electrons.

It was then obvious from analysis and experiment that the energy of a quantum must be equal to the frequency of the photon times Planck's constant. This relation must also be reconciled with two other observations. The first is the Doppler shift, and we will consider first the case of radial velocity for simplicity here. We know that, for radial velocity toward the observer, the Doppler shift is equal to c divided by the initial component velocity of the light. The second consideration is the alternate expression for photon energy, in terms of mass equivalent, given as the mass times c^2 . We can combine these relations to show that the frequency of a photon is equal to its mass times c^2 divided by Planck's constant. Now, Planck's constant and c^2 are by definition constants in any given inertial frame of reference, yet the frequency changes when going from one frame of reference to the next according to the Doppler

shift relation. This would imply a velocity dependence to the mass of a photon. This seems to be in contradiction to the previous two sections which indicated that there is no dependence of mass on velocity. The resolution lies in the fact that a photon actually has no mass at all. All of the contributions to mass in the case of a photon are due to effective mass equivalence.

When Planck made his observations, he was necessarily observing the energy of a photon traveling at c in his frame of reference. Even so, his expression for the energy represents the total energy of the absorbed photon. It does not make sense to speak of the energy of any given velocity component of the photon, as the entire photon is either absorbed or emitted as a whole, not as incremental components. We can also speak of the momentum of the photon, and in this case, we can consider the momentum of each velocity component. For the time being, let us define the reference frame of the source and the observer to be the same, thus, we are interested in the velocity component traveling at c . We can use Planck's energy relation to define a unit of momentum. At the point at which Planck is observing the photon, the momentum is given by the energy divided by the velocity, which is of course always c at the point of observation. We can assign this value to all points along the photon wave for the following reasons: 1) the photon wave has the same frequency at all points (the wavelength changes with the velocity), 2) an interaction may occur at any point along the photon wave, depending on the relative velocity of the observer, and 3) when the photon is absorbed, the entire photon wave is absorbed at that point.

The momentum of the photon at any point may alternatively be defined as its mass times its velocity at that point. Since we have defined the momentum of an emitted photon to have the same value along its entire length, the effective mass at each point must change to accommodate this, just as the wavelength changes to keep the frequency constant. In fact, it can be shown quite simply that the product of the mass and wavelength at any point along the photon wave is equal to a fixed quantity given by Planck's constant times the frequency divided by c . A decrease in component wavelength is matched by an equivalent increase in effective component mass. Thus, the effective mass at a slower velocity component would be greater than that at a higher velocity component. The ratio of the various component masses to the mass of the c velocity component is given by an equation similar to that for the Doppler shift--the mass shifts by c divided by the velocity of the component of interest.

Now imagine an observer moving toward this component at a velocity that would give the component a speed of c in his frame of reference. To this observer, the momentum of that component of the photon is its effective mass times its velocity in his frame. The energy of the photon then equals the momentum times the velocity, and also, according to Planck's rule, equals Planck's constant times the shifted frequency. Combining these two relations for the energy, we can derive an expression for the shifted frequency of the photon absorbed by the moving observer. The shift in frequency is equal to c divided by the initial velocity of the component of interest. In chapter three we derived the frequency shift by using the same treatment of waves and frequencies that Doppler used in his derivations. In this section we proceeded by analyzing the energy difference between the emission of a photon in one reference frame and its absorption in another. In both cases, approaching the problem from two different avenues, we obtain the same result. A similar treatment produces the Doppler shift for perpendicular incidence as derived in chapter four.

Referring once again to figure 4-2 and the Pythagorean theorem, we see that the initial velocity component of light required by an observer with perpendicular motion is equal to the square root of the sum of the squares of c and the observer's velocity. The shift in the effective mass of this component is then given by c divided by the velocity of this component. To first approximation for speeds much less than c , this shift is equal to one over the gamma factor for the observer's velocity. Thus the energy of the photon absorbed by the observer is equal to the energy of the emitted photon divided by gamma, and, by Planck's rule, the shifted frequency is equal to the initial frequency divided by gamma as well. Thus, using Planck's observations, and the rules for momentum and energy, we have derived the Doppler shift formulas for parallel and perpendicular incidence expressed in chapter four. Combining these two relations using simplifying assumptions valid for speeds much less than c , we arrive at the same equations derived in special relativity. This approximation to the relativistic form of the Doppler shift holds only in limited cases, but these cases account for all experimental observations made to date. Limiting the observer's velocity to be much less than c , the effects due to any velocity in the radial or parallel direction will swamp the perpendicular velocity effects, and the equation will reduce to the standard Newtonian Doppler equation. If the velocity is purely in the perpendicular direction, then the frequency shift is equal to one over gamma, and only the so-called "relativistic time-delay" Doppler effect appears. Only when the velocity approaches c will the

approximation to the relativistic formula fail to hold, and then, depending on the angle between the source and the observer, the difference can become quite significant. The only sources which we know of with respect to which we have velocities great enough to reflect the difference between RCM theory and relativity theory are very remote galaxies. Unfortunately, we must use the very Doppler calculations we are trying to test in order to determine their velocity. With no independent means to test their velocity, the discrepancy between the two theories at such speeds cannot, as yet, be tested.

If we imagine photon's as they exist in relativity theory, each traveling at a constant speed of c , and with an associated energy of mc^2 , then we must come to the conclusion that a photon of a given frequency is fundamentally different than another photon of a different frequency. The effective mass of each of these photon's is unique. One is forced to wonder why or how there could be so many unique types of photons, one for each possible frequency we may imagine. Since, for any two frequencies, no matter how close their values, we can imagine another frequency midway between these two, there is also a unique photon type associated with this frequency. It is easy to see then that, in the relativistic model, the number of types of photons is infinite. This is an uncomfortable notion. One would imagine that a photon is a photon is a photon. A high energy electron is still an electron, and there is only one type of electron. The same is true of neutrons, muons, protons, and any other particle one wishes to consider. And yet we are to believe that there are an infinite number of types of photons.

Now, consider a photon as it exists in RCM theory. For purposes of illustration, the photon may be visualized at some point in time as pictured in figure 7-5. At any point along this wave, we can measure the wavelength. We have already seen how a particular wavelength can be alternatively expressed as an equivalent mass, thus, the shorter wavelengths correspond to greater mass. It is also true that the shorter wavelengths represent lower velocity components of the photon. Thus we have the situation whereby, for any given photon at any given energy, the more "massive" components travel at a slower speed than the less massive components. This is very intuitively comforting. One would expect that, for a given energy, we would be able to push a Volkswagen faster than a Mack truck. Now, imagine that every photon has the exact same structure. How do we obtain the infinite range of achievable frequencies? Consider a point on the photon with a given mass or a given wavelength. If we generate the photon with enough energy so that that component is traveling at a speed of c , the photon's energy will be given by mc^2 . Now, suppose we create a photon with more energy. In this case, we can force a "more massive" component of the photon to obtain a speed of c . A Mack truck has a much higher energy engine than a Volkswagen, thus it is able to accelerate a much more massive vehicle to any given speed. The measured energy of our new photon will be greater than the first, since the mass in the energy equation will be greater. Thus, all photon's are equal in form, but the energy supplied at the time of their creation determines the relative velocity of each component, in much the same manner that a car of a given mass moves at a greater speed when one applies more energy by pressing on the "accelerator."

Alternatively, we may state that if we supply a greater energy in creating the photon, a smaller wavelength component will achieve a speed of c . Since frequency is equal to c divided by the wavelength of the component traveling at that velocity, and the wavelength is smaller, the frequency will, of course, be greater. Starting with our one, universal photon, we can vary the energy supplied to create it over an infinite number of values, and thus create an equally infinite number of values of frequency. In RCM theory, a high energy photon is just a high energy photon. There is only one type of electron, one type of muon, and one type of photon.



Figure 7-5 *A graphical representation of a photon in RCM theory. The faster components have longer wavelengths, and the energy supplied by the source determines the velocity of each point and also the frequency of the photon.*

Even with the simple analysis of photon energy provided above, we might still ask "Why is a photon so fundamentally different than any of the elementary particles in the way it carries energy?" A particle in motion has

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energy equal to its rest mass plus its kinetic energy, and its kinetic energy changes according to its velocity or the velocity of any observer with respect to it. While an absorbed photon changes its energy by virtue of a Doppler shift due to the observer's relative motion with respect to its source, we have seen repeatedly that there appears to be no distinction between its kinetic energy and its inherent energy. Both energies appear to be rolled up intimately in the nature of the photon, and to be also inseparable one from the other. However, this may be due to a fallacy in the way we view both the "rest mass" of a photon and the nature of kinetic energy.

First, remember that a photon, which is considered pure energy, is affected in a gravitational field in the same manner as mass energy. We also illustrated in chapter seven that kinetic energy itself is affected in a gravitational field when we derived the anomalous perihelion advance of Mercury. We also know from Planck's studies of photons that photon energy can be expressed as kinetic energy due to an equivalent mass similar to the way in which a physical mass strikes an object with kinetic energy due to its motion. This equivalence of all forms of energy leads to an interesting question--does kinetic energy itself possess kinetic energy? From the examples above, it would appear that the answer should be yes. But why don't we see this effect in our everyday experience?

Imagine a large automobile traveling at a high velocity. This auto has kinetic energy equal to one-half its mass, which is relatively large, times its velocity squared. The result is a large value of kinetic energy, which we feel if the car hits us from behind. But how much additional impact is supplied by the additional kinetic energy of the mass equivalent of the auto's kinetic energy. We can express the auto's kinetic energy as an equivalent mass by dividing by c^2 , thus the equivalent mass of the kinetic energy is equal to the mass of the car times its velocity squared divided by two times c^2 . The kinetic energy of this equivalent mass is then equal to one-half this mass times the velocity squared--the same formula used for the automobile itself, but with a much smaller value for the mass. This additional kinetic energy would, in fact, be almost negligible. Even if we could drive our car at one-percent the speed of light, or 1,860 miles per second, the revised kinetic energy would be the same as the Newtonian kinetic energy for a car weighing less than one-quarter pound more than a typical car. This is a very small difference indeed, and it is small because, despite the relatively large mass of the automobile, the velocities with which we are dealing are much smaller than c .

Now consider a photon traveling at a speed of c in the reference frame of its source. If this were a typical particle, its kinetic energy would be equal to one-half its mass times its velocity squared, or one-half its mass times c^2 . The equivalent mass of this energy would be equal to this energy divided by c^2 , or simply one-half the rest mass of the photon. This is a much larger equivalent mass than we saw with the automobile example. The kinetic energy of the equivalent mass of this kinetic energy term is then one-half the equivalent mass times c^2 , or one-fourth the photon's rest mass times c^2 . Thus the additional kinetic energy supplied by the kinetic energy itself is half as large as that attributed to the photon's rest mass. This type of energy is too large to be ignored. But that is not all.

The second kinetic energy term derived above has an equivalent mass equal to one-fourth the rest mass of the photon. Thus the kinetic energy of this equivalent mass is also too large to ignore. In fact, when we add up all the additional kinetic energy terms for a photon, we find that its total kinetic energy is equal to its rest mass times c^2 , twice the Newtonian value. The total energy of the photon would then be equal to two times its rest mass times c^2 . If we therefore assume the "rest mass" of a photon to be one-half the value proposed by Planck and Einstein, we find a photon in which we can distinguish between its inherent or rest energy, and its kinetic energy, while still maintaining the characteristic total energy experienced by Planck and Einstein. Defining a photon in this manner, with one-half the rest mass of a relativistic photon, all of the previous analysis provided in this chapter still retains its validity. We have simply attributed one half of the energy of a photon to its rest mass, and the second half to the sum of all its kinetic energy terms.

While this may seem like a purely semantic argument, it carries with it two important points. First, since all energy is equivalent, then kinetic energy should possess additional kinetic energy due to its equivalent mass and its velocity. If such were not the case, there would be something fundamentally different about kinetic energy in comparison to other forms of energy, in contradiction to the ease with which we can convert from one form of energy to another--say from gravitational potential energy to kinetic energy and then to heat energy as we drop an object from a tower. Second, the presumption of a photon with unique and separable rest and kinetic energies provides us with yet another derivation of the Doppler shift utilizing energy considerations and Galilean transformations.

Imagine a source of light centered at the origin of one reference frame which is moving away from an observer at the center of another reference frame at a constant velocity, as in frame A of figure 7-5. If the observer detects a photon at a given frequency, it will have a rest energy given by its mass times c^2 , where, again, this mass is equal to one-half the rest mass of a relativistic photon. The component which the observer will ultimately detect must have a velocity with respect to the source of c plus the separation velocity of the reference frames. The kinetic energy of a component traveling at this velocity is higher than that of the same component traveling at c proportional to the relative velocity of the reference frames. A Galilean transformation of the total energy of the photon emitted from the source, as seen from the observer's frame of reference, is equal to the detected or observed energy times one plus the velocity of the reference frames divided by c . Inverting this relation yields the observed frequency as compared to the emitted frequency. The observed frequency is equal to the source frequency times c divided by c plus the separation velocity, just as we saw in chapter three and the first part of this chapter. This is the equation for Doppler shift due to motion along the line joining the source and the observer, but what about motion perpendicular to this line?

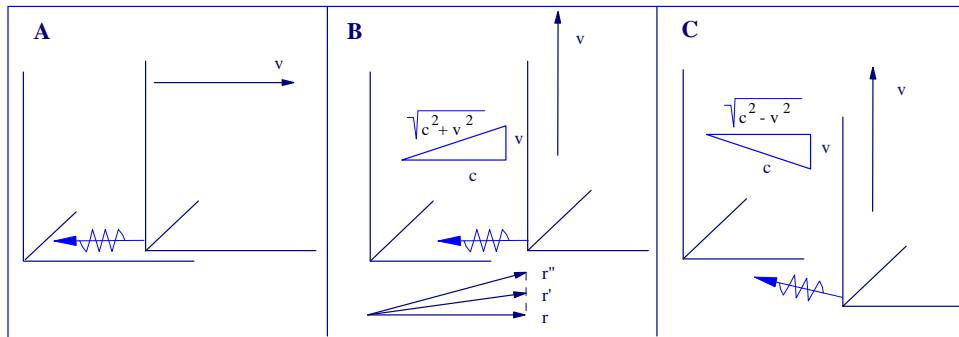


Figure 7-5 The geometry of an analysis of the Doppler shift utilizing kinetic energy considerations.

If we begin with the origins of the reference frames initially separated by some fixed distance, and the source reference frame moving up the page as in frame B of figure 7-5, we have the case of perpendicular motion. As the figure shows, there will be an increase in the radial separation distance between the origins of the reference frames related by the Pythagorean theorem to the velocity of the source and c . If the observer detects a photon with a given frequency, he can convert it to the frequency of the source via a Galilean transformation by multiplying by the square root of one plus the velocity squared, with the velocity expressed as a fraction of c , where this term describes the change in radial distance between the origins of the reference frames. Inverting this expression produces the result that the observed frequency is equal to the source frequency divided by this factor. This is identical to the expression developed in chapter four for the Doppler shift of light which is actually perpendicularly incident on a moving observer.

Frame C of figure 7-5 illustrates the case of light which will appear perpendicularly incident upon the observer. The only difference between frame B and C in this analysis is that the distance between the origins of the reference frames is decreasing rather than increasing. Thus we can use the same expression derived above except we subtract the velocity squared term rather than adding it. This is the result derived in chapter four for the Doppler shift of light which appears perpendicularly incident upon an observer.

From this simple energy analysis, considering the rest energy and kinetic energy of the photon as separable entities, based on a rest mass equal to one-half the relativistic value, we have derived the three relations for Doppler shift derived in chapters three and four, and again at the start of this section. This same analysis produced a photon with energy equal to Planck's constant times the frequency, in keeping with previously derived results. Additionally, we have unified kinetic energy with other forms of energy, such as mass energy, and reduced the number of photon types from an infinite number to only one. We now turn to one more so-called test of relativity theory--the time required for a light signal to complete a trip which brings it very close to a large gravitating mass.

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THE TIME DELAY OF LIGHT

Imagine a photon in free fall in a solar-radius grazing path as it passes the sun. We have seen how a photon falling towards the sun will have its frequency red-shifted (toward the blue or higher frequencies in this case). This may be thought of as due to increasing the effective mass of the photon, or, conversely, as "bunching up" the wavelength of the photon, thus increasing the frequency. Using the results obtained for the gravitational red-shift in chapters four and six, we see that the frequency increases proportionally to the increase in gravitational potential. Chapter three illustrated the relation between the initial velocity of light and the shift in frequency. Combining these two effects, we can determine the speed at which this shorter wavelength should be traveling for a given photon. Since shorter wavelengths are accompanied by slower speeds in a given photon, the speed of the photon will be reduced proportionally to the shift in frequency, or, alternatively, to the shortening of the wavelength. Thus the new speed of the component initially at c will be reduced by a proportion identical to that for the gravitational red-shift. Of course, the photon doesn't actually do anything to keep its speed or its frequency at the correct value for its surroundings. The laws governing the conservation of energy take care of these effects automatically. In fact, we could get by without actually describing a change in frequency or a change in velocity of photon components, and simply keep track of the energy equations at all times instead. That is, in fact, what we are actually doing. But we have seen earlier in this chapter that the observed frequency of a high energy photon is higher than that of a low energy photon, and that the velocity of any given component of a photon is proportional to its energy as well, so speaking of frequency and velocity is equivalent to speaking of energy. Thus the descriptions I have used describing a photon's efforts at "slowing itself down" and "trying to keep its frequency constant" should not be taken literally. There is no need to assume the photon has any conscious awareness of its existence or its surroundings.

It is straightforward to calculate the effect of the changing velocity described above on the travel time of a photon falling towards the sun. Imagine that we are going to measure the travel time of a light signal making a round trip between the earth and mars at superior conjunction. Superior conjunction is when mars is on the opposite side of the sun from earth, so that this light signal would just graze the sun on each passage. We would expect the round trip travel time to be longer for the slower velocity component than would be the same trip made at a velocity of c . This is actually the case, as the derivation in Appendix C shows. When the effects of the decrease in velocity are calculated for the entire round trip, we find the photon experiences a time delay of roughly 125 microseconds (125×10^{-6} sec.). This is equivalent to an uncertainty in the distance to mars of one-half of this delay times c , or 19 kilometers.

The measurement alluded to above has been carried out. Irwin Shapiro first calculated the time delay of a solar grazing light ray based on relativity considerations in 1961, but at the time, there was no way to test his predictions. However, by 1964, radar ranging of the distance to various planets was becoming practical and accurate, so Shapiro published his predictions in *Physical Review Letters*. While difficult in practice, the test for such an effect is in theory, quite simple. The round trip travel time of a light or radar signal traveling at the speed of c can be used to indicate the distance to a planet by multiplying the time by c and dividing by two. Since the orbit of a planet can be calculated very accurately, its position is well known at all times. Therefore, if we send a signal to a planet when it is at superior conjunction (on the opposite side of the sun from earth), and calculate its distance based on the round trip travel time of the signal, an error in the distance obtained would be attributable to a change in the velocity of light. If the light signal slowed down, the planet would appear to be further away than it actually is, while if the light signal speeded up, the planet would appear to be closer than we know it to be.

This test was not actually performed until 1967, and it was performed by ranging to the planet Mercury while at superior conjunction. The results were in agreement with Shapiro's derivations from Einstein's equations to within twenty percent. While these results are good enough to show that there is indeed a time delay, a more accurate test is required. In 1969, the same test was performed using the Mariner 6 and 7 satellites, whose main mission was to study the planet Mars. The use of satellites and their controlled, directional transmitters improved the accuracy of the measurements greatly. In fact, the results came obtained were in agreement with Shapiro's calculations to within three percent.

Returning to our original example, then, we would like to know how closely the relativistic prediction of Shapiro agrees with the RCM prediction. The Shapiro time delay in our first example comes out to be 250 microseconds, exactly twice that which we calculated above. How can we account for the additional 125 microseconds? The answer lies again in the gravitational red-shift. Recall that our formula for the gravitational red-shift compares the frequency as measured (generated) at one point in the gravitational field with the frequency as measured (received) at another point in the field. If the experiment is carried out over a short distance, as in the Pound-Rebka experiment, then the gravitational field remains roughly constant throughout the experiment. In this case, we can ignore the effects of the gravitational field on clocks, as all clocks used in the experiment are basically collocated and will be affected to the same degree. If we extend our experiment to cover a much larger distance, say from one solar radius to the earth, then we must take the effect of the extreme change in the gravitational field on our clocks into account.

We demonstrated in chapter four how a clock close to the sun would run more slowly than one near the earth due to the effects of the increased gravitational potential. If we use this clock to measure frequency locally (say near the sun), then we would obtain a different value than we would had we used a clock farther away (say at the earth). Thus when we speak of the gravitational red-shift, we are speaking of the difference in frequencies *as measured locally* using local clocks. Since we are going to use only an earth-based clock to measure the time delay, we must transpose each "locally" measured frequency into an equivalent frequency had it been measured using our earth-frame clock, the "proper" time of chapter four. Thus, if a frequency is observed to have one value using a clock close to the sun, that frequency would be lower using our earth-based clock, since our earth-based clock runs faster than the local clock near the sun. In a similar fashion, what appears as a particular velocity using clocks near the sun would appear as a slower velocity using clocks calibrated to match those near the earth.

Note that the effect of slowing of "local" clocks does not require that one actually measure a quantity such as velocity with a physical clock. A photon component traveling at c , for example, due to its nature, must cover a specific distance equal to its wavelength in a specific time in its frame of reference, that time being equal to one over the frequency. If this time is measured with longer units, then c must be slower such that the same distance is covered in a given unit of time measured with that clock. It is obvious (Appendix C) that this effect will be of the identical magnitude as the velocity shifting effect above, again directly related to the gravitational potential of the sun. When this effect is integrated over the entire path of the photon, the result is an additional delay of 125 microseconds. Thus the total time delay amounts to 250 microseconds. This equates to an uncertainty in the distance to Mars at superior conjunction of about 38 km, or a total round-trip uncertainty on the order of 76 km. This is in complete agreement with the results of Shapiro's ranging experiments, which have been improved upon more recently to provide an accuracy of 0.1%.

Relativity theory accounts for the time-delay by combining the "length-contraction" and "time-dilation" effects of the theory. In RCM theory, the first half of the effect is due to the equivalence principle, which results in the component of the photon initially traveling at c slowing itself to some lower velocity. The second half is due to the effect which the gravitational red-shift has on measurements of velocity in the vicinity of the sun as opposed to great distances from the sun, or the effects of using "local" versus "proper" clocks--the slower speed measured with a slower clock is twice delayed when its travel time is measured only against a proper clock sitting stationary on the earth.

Having seen how important energy considerations are when dealing with gravitational effects, we will next tackle the two remaining great "tests" of the general theory of relativity--the anomalous advance in the perihelion of Mercury, and the deflection of starlight grazing the sun.